King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer Semester 2014-2015
Final Exam, Wednesday August 12, 2015.

Name:

ID#:

Instructions:

1. This exam consists of **nine** pages, including this page and the final reference sheet, containing **six** questions.
2. You have to answer all **six** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **not equally weighed**.
5. This exam is out of **100** points, although you can earn a maximum **105** points.
6. You have exactly **150** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

Question Number	Maximum # of Points	Earned/Deducted Points
I	**20**	
II	**25**	
III	**15**	
IV	**15**	
V	**15**	
VI	**15**	
Total	**100**	

I. (20 points) Choose the correct answer from the following choices. Note that the number of choices is not equal for all questions.

1. The negation of the proposition: "Ahmad and Salim are present" is
   - (a) Ahmad and Salim are absent.
   - (b) Ahmad or Salim is absent.
   - (c) Ahmad is present and Salim is absent.
   - (d) Ahmad is absent and Salim is present
   - (e) all of the above.

2. On the island of knights and knaves you encounter two people. *A* and *B*. Person *A* says, "*B* is a knave." Person *B* says, "At least one of us is a knight." Then,
   - (a) *A* is a knave, *B* is a knave.
   - (b) *A* is a knight, *B* is a knight.
   - (c) *A* is a knight, *B* is a knave.
   - (d) *A* is a knave, *B* is a knight.
   - (e) one of them cannot be determined for sure whether he is knave or knight.

3. Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," and A(x, y) the predicate "x has asked y a question," where the domain consists of all people associated with your school. Then, the statement "Some student has never been asked a question by a faculty member" is represented by
   - (a) $\exists x\left(S(x)\wedge\forall y\left(F(y)\rightarrow\neg A(y,x)\right)\right)$
   - (b) $\forall x\left(S(x)\rightarrow\forall y\left(F(y)\rightarrow\neg A(y,x)\right)\right)$
   - (c) $\forall y\left(\exists x S(x)\wedge\left(F(y)\rightarrow\neg A(y,x)\right)\right)$
   - (d) $\exists x\left(S(x)\wedge\forall y\left(F(y)\wedge\neg A(y,x)\right)\right)$
   - (e) none of the above.

4. Suppose $A = \{x,y\}$ and $B = \{x,\{x\}\}$. Then, it is true that
   - (a) $x \subseteq B$.
   - (b) $\varnothing \in P(B)$.
   - (c) $\{x\} \subseteq A - B$.
   - (d) $|P(A)| = |P(B)|$.
   - (e) more than one answer above is true.

5. Which of the following statements is a true statement
   - (a) $(A - C) - (B - C) = A - B$.
   - (b) $\overline{A \cup \overline{B} \cup \overline{A}} = \overline{A}$.
   - (c) If $A \cup C = B \cup C$, then $A = B$.
   - (d) If $A \cap C = B \cap C$, then $A = B$.
   - (e) none of the above.

6. The sum $2 - 4 + 8 - 16 + 32 - \ldots + 2^{27} - 2^{28}$ is
   (a) $2 + 2\times2^{29}$
   (b) $2 - 2\times2^{29}$
   (c) $(2/3) + (2^{29}/3)$
   (d) $(2/3) - (2^{29}/3)$
   (e) none of the above.

7. $\bigcap_{i=1}^{\infty}\left[-1+\frac{1}{i}, 1+\frac{1}{i}\right]=$
   (a) $(-1,1)$.
   (b) $[-1,1]$.
   (c) $(-2,2)$.
   (d) $[0,1]$.
   (e) none of the above.

8. $f:\Re\to\mathbf{Z}$ where $f(x)=\lceil 2x-1\rceil$ is
   (a) not a function
   (b) one to one but not onto
   (c) onto but not one to one
   (d) one to one and onto
   (e) neither one to one nor onto

9. The sequence $\{a_n\}$ that is a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ is
   (a) $a_n = n^2 4^n$
   (b) $a_n = (-4)^n$
   (c) $a_n = 1$
   (d) $a_n = 2^n$
   (e) $a_n = n\,4^n$

10. $\sum_{i=2}^{100}\sum_{j=1}^{i}(ij)=$
   (a) 12920425.
   (b) 12920424.
   (c) 505000.
   (d) 338349
   (e) none of the above.

II. (25 points) Basic Counting and the pigeonhole principle.

1. (5 points) How many bit strings of length seven either begin with two 0s or end with three 1s?

$2^5+2^4-2^2$

2. (7 points) The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? (Note that the name of a variable may contain fewer than eight characters.)

$$\sum_{j=0}^{7}(26+26+1)*(26+26+10+1)^j$$

3. (5 points) A bowl contains 10 red balls and 10 blue balls. A man selects balls at random without looking at them.
a) How many balls must he select to be sure of having at least three balls of the same color?
b) How many balls must he select to be sure of having at least three blue balls?

$$3=\left\lceil\frac{N}{2}\right\rceil \rightarrow N=5$$

4. (8 points) An arm wrestler is the champion for a period of 75 hours. (Here, by an hour, we mean a period starting from an exact hour, such as 1 p.m., until the next hour.) The arm wrestler had at least one match an hour, but no more than 125 total matches. Show that there is a period of consecutive hours during which the arm wrestler had exactly 24 matches.

Let x_i denote the no. of matches the wrestler had played after the end of the i-th hour for $1 \leq i \leq 75$, where $1 \leq x_i \leq 125$. This also implies that $25 \leq x_i + 24 \leq 149$. Thus 150 integers $x_1, x_2, \ldots, x_{75}, x_1 + 24, x_2 + 24, \ldots, x_{75} + 24$ are between 1 and 149, and the Pigeonhole Principle implies that $\exists\ i \neq j \ \ni\ x_i + 24 = x_j$, i.e. the wrestler played exactly 24 matches between the end of i-th and j-th hour.

(15 points) Permutations, Combinations and the Binomial Coefficients.

1. (5 points) A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

C(40,17)

2. (5 points) How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?

$$C(45,3)*C(57,4)*C(69,5)$$

3. (5 points) What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

$$C(17,8)*3^8*2^9$$

III. (15 points) Introduction to Discrete Probability.

1. (5 points) What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?

$$\frac{4 * \binom{13}{5}}{\binom{52}{5}}$$

2. (5 points) Suppose that 100 people enter a contest and that different winners are selected at random for first, second, and third prizes. What is the probability that Michelle wins one of these prizes if she is one of the contestants?

$$\frac{C(3,1) * 99 * 98}{100 * 99 * 98}$$

3. (5 points) What is the probability of rolling a total of 9 when three dice are rolled?

$$tedious$$

IV. (15 points) Probability Theory.

1. (8 points) Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has at least two girls?

$$1-C(5,1)*.51^4*.49-C(5,0)*.51^5$$

2. (7 points) What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

$$\frac{1}{2}^{4}$$

V. (15 points) Advanced Counting Techniques.

1. (a) (5 points) Find a recurrence relation for the number of bit strings of length *n* that do not contain three consecutive 0s.
   (b) (2 points) What are the initial conditions?

$$a_n = a_{n-1}+a_{n-2}+a_{n-3}$$
$$a_1 = 2$$
$$a_2 = 4$$
$$a_3 = 7$$

2. (8 points) Solve the following recurrence relation, together with the initial conditions given.

$$a_n = 6a_{n-1} - 8a_{n-2}\ for\ n \geq 2, a_0 = 4, a_1 = 10$$

$$x^2 \text{ -6x+8 = 0 -> (x-4)(x-2) = 0}$$
$$an = 2^n*\alpha+4^n*\beta$$

Some Useful Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ where } a \neq 1 \quad , \quad \sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ where } |a| < 1,$$

$$\sum_{i=1}^{\infty} i a^{i-1} = \frac{1}{(1-a)^2} \text{ where } |a| < 1$$

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$((p) \wedge (q)) \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws